$$H = \begin{bmatrix} I & I & I & \cdots\cdots & I \\ I & \sigma^1 & \sigma^2 & \cdots\cdots & \sigma^{p-1} \\ I & \sigma^2 & \sigma^4 & \cdots\cdots & \sigma^{2(p-1)} \\ & & & & \\ I & \sigma^{r-1} & \sigma^{2(r-1)} & \cdots\cdots & \sigma^{(r-1)(p-1)} \end{bmatrix}$$

FIG.1

FIG.2

FIG.3

420   f (401)   410

information   parity
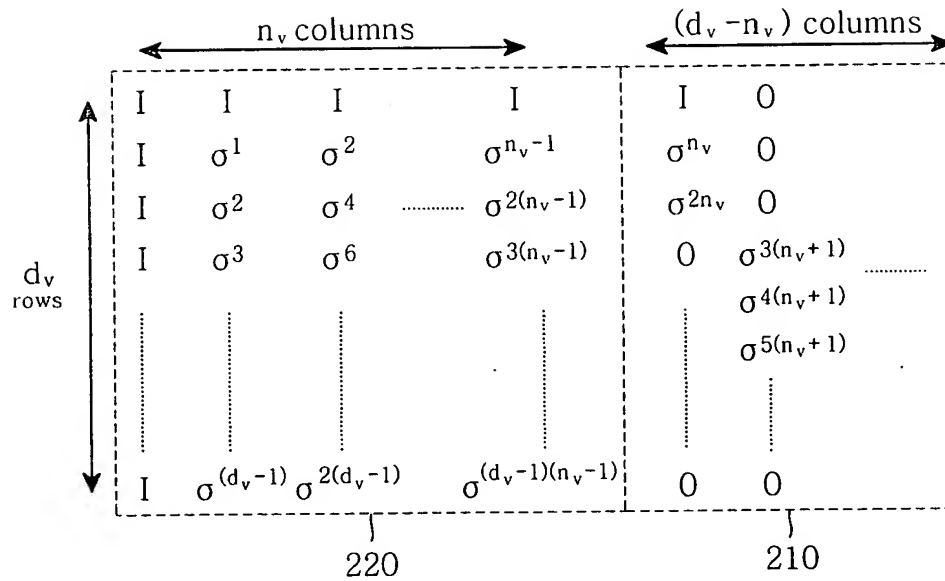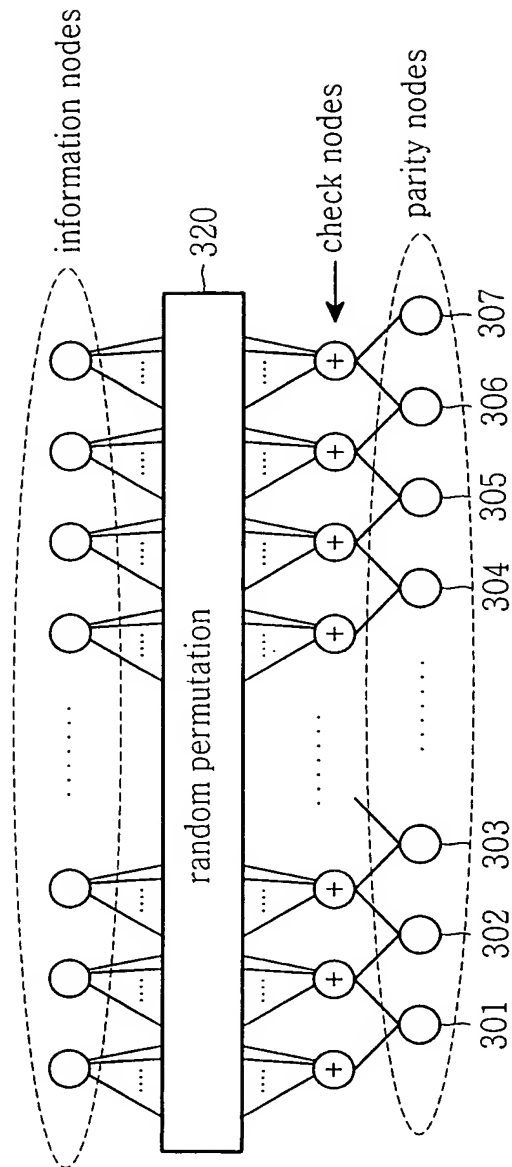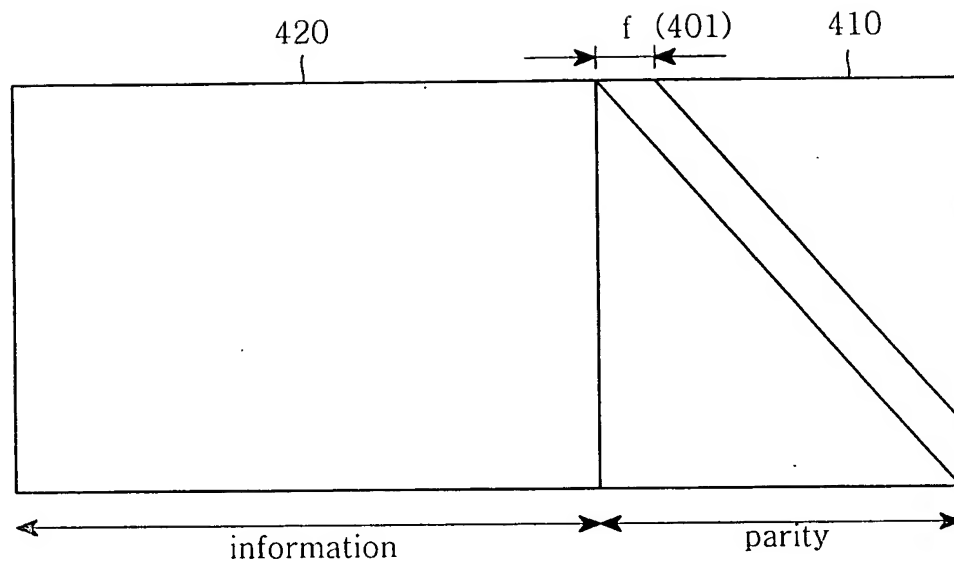
FIG.4
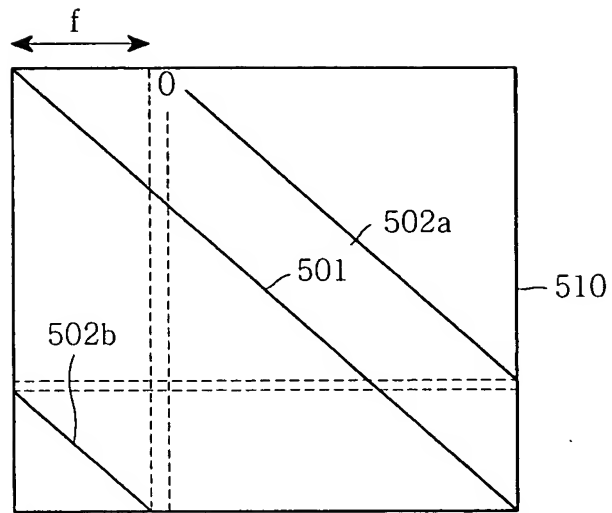
FIG.5

FIG.6

0	0	0	0			0		0		
0	1	2	3			6		11		
0	2	4	6			12		22		
0	3	6	9			18		33		
0	4	8	12			28			48	
0	5	10		20		35			60	
0	6	12		24		42			72	
0	7	14		28			56		2	
0	8	16		32			64		15	
0	9	18		36			72		28	
0	10	20		50			1			51
0	11	22		55			10			65
0	12	24		60			19			79
0	13	26		65			41			
0	14	28		70			51			

Example of H_d with irregular distribution of $d_v = 15$ (p=89)

FIG.7

802a 803a

804a

$$
\begin{array}{ccc|ccc|ccc|ccc}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hline
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\end{array}
$$

801a

801 — 0   0   1   0 — 803
802 — 1   0   0   1 — 804
        0   1   0   1
        0   0   1   0

FIG.8

fsub-matrix columns(pxf columns)

$$H_P = \begin{bmatrix} \sigma^{j_0} & & & \sigma^{j_1} & \\ & \sigma^{j_2} & & & \sigma^{j_3} & \\ & & & \sigma^{j_{2(r-f-1)}} & & & \sigma^{j_{2(r-f-1)+1}} \\ \sigma^{j_{2(r-f)+1}} & & & \sigma^{j_{2(r-f)}} & \\ & & \sigma^{j_{2(r-1)+1}} & & & \sigma^{j_{2(r-1)}} \end{bmatrix}$$

FIG.9

START

SET PARITY SYMBOL
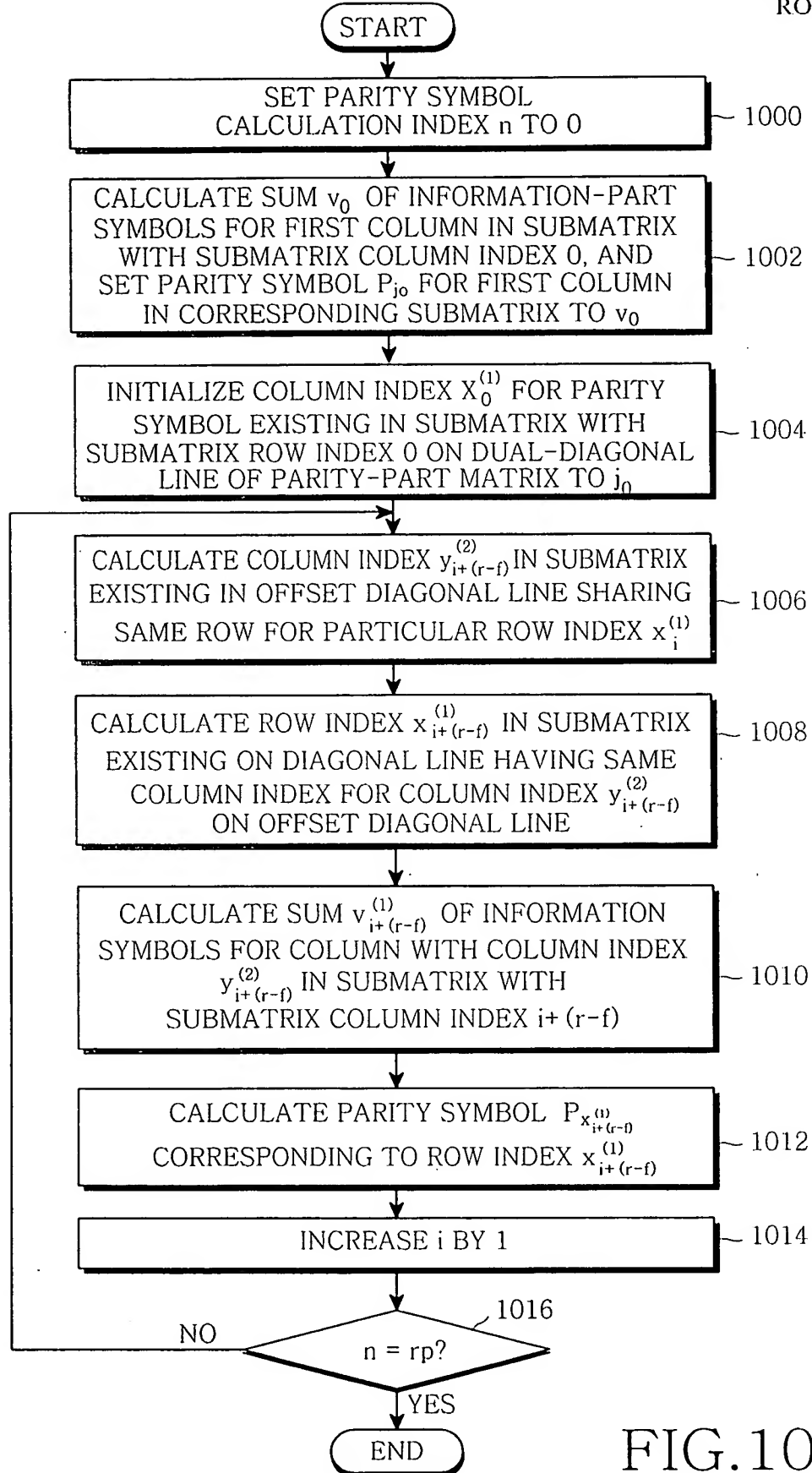CALCULATION INDEX n TO 0 — 1000

CALCULATE SUM v_0 OF INFORMATION-PART
SYMBOLS FOR FIRST COLUMN IN SUBMATRIX
WITH SUBMATRIX COLUMN INDEX 0, AND
SET PARITY SYMBOL P_{j_0} FOR FIRST COLUMN
IN CORRESPONDING SUBMATRIX TO v_0 — 1002

INITIALIZE COLUMN INDEX $X_0^{(1)}$ FOR PARITY
SYMBOL EXISTING IN SUBMATRIX WITH
SUBMATRIX ROW INDEX 0 ON DUAL-DIAGONAL
LINE OF PARITY-PART MATRIX TO j_0 — 1004

CALCULATE COLUMN INDEX $y_{i+(r-f)}^{(2)}$ IN SUBMATRIX
EXISTING IN OFFSET DIAGONAL LINE SHARING
SAME ROW FOR PARTICULAR ROW INDEX $x_i^{(1)}$ — 1006

CALCULATE ROW INDEX $x_{i+(r-f)}^{(1)}$ IN SUBMATRIX
EXISTING ON DIAGONAL LINE HAVING SAME
COLUMN INDEX FOR COLUMN INDEX $y_{i+(r-f)}^{(2)}$
ON OFFSET DIAGONAL LINE — 1008

CALCULATE SUM $v_{i+(r-f)}^{(1)}$ OF INFORMATION
SYMBOLS FOR COLUMN WITH COLUMN INDEX
$y_{i+(r-f)}^{(2)}$ IN SUBMATRIX WITH
SUBMATRIX COLUMN INDEX i+ (r−f) — 1010

CALCULATE PARITY SYMBOL $P_{x_{i+(r-f)}^{(1)}}$
CORRESPONDING TO ROW INDEX $x_{i+(r-f)}^{(1)}$ — 1012

INCREASE i BY 1 — 1014

1016

NO ← n = rp?

YES

END

FIG.10

Example of H_p by lifting the generalized dual-diagonal matrix (r=15, f=7, p=89)

FIG.11

```
                    ┌─────────┐
                    │  START  │
                    └─────────┘
                         │
                         ▼
        ┌──────────────────────────────────┐
        │  INITIALIZE  q_mn^(0) = ℓ_n^(0)   │──1200
        └──────────────────────────────────┘
                         │
                         ▼
        ┌──────────────────────────────────┐
        │  UPDATE CHECK NODE MESSAGE        │──1202
        └──────────────────────────────────┘
                         │
                         ▼
        ┌──────────────────────────────────┐
        │  UPDATE VARIABLE NODE             │──1204
        │  MESSAGE AND LLR                  │
        └──────────────────────────────────┘
                         │
                         ▼
        ┌──────────────────────────────────┐
        │  HARD DECISION ON LLR  x̂          │──1206
        └──────────────────────────────────┘
                         │
                         ▼
                     1208
        YES        ◇────────────◇
   ┌───────────────│  Hx̂ = o ?   │
   │               ◇────────────◇
   ▼ 1216               │ NO
┌──────────┐            ▼      1210
│  STOP    │        ◇──────────────◇       NO
│ DECODING │        │ PREDETERMINED │──────────────┐
└──────────┘        │  ITERATION?   │              │
   │                ◇──────────────◇            1212│
   │                     │ YES          ┌──────────────────┐
   │                     ▼              │  INCREASE        │
   │        ┌────────────────────────┐  │ ITERATION COUNTER│
   │        │ DECODING FAILURE PROCESS│──1214         └──────────────────┘
   │        └────────────────────────┘
   │                     │
   └──────────┬──────────┘
              ▼
          ┌───────┐
          │  END  │
          └───────┘
```

FIG.12

$- \mathbf{H_d}$

0	0	0	0		0	0
0	1	2	3		6	11
0	2	4	6		12	22
0	3	6	9		18	4
0	4	8	12		28	19
0	5	10		20	6	2
0	6	12		24	13	14
0	7	14		28	27	4
0	8	16		3	6	17
0	9	18		7	14	1
0	10	20		21	3	24
0	11	22		26	12	9
0	12	24		2	21	23
0	13	26		7	14	
0	14	28		12	24	

FIG.13A

$- H_P$

```
15                    9
   11                   13
      11                  26
        17                 22
          26                17
            ·25              ·15
              26                7
                3              15
        7        9
      16          28
        20          24
          18          ·18
            21          0
              20 ·        18
                5          28
```

FIG.13B

FER & BER performance (n=0.8K, R=1/2)
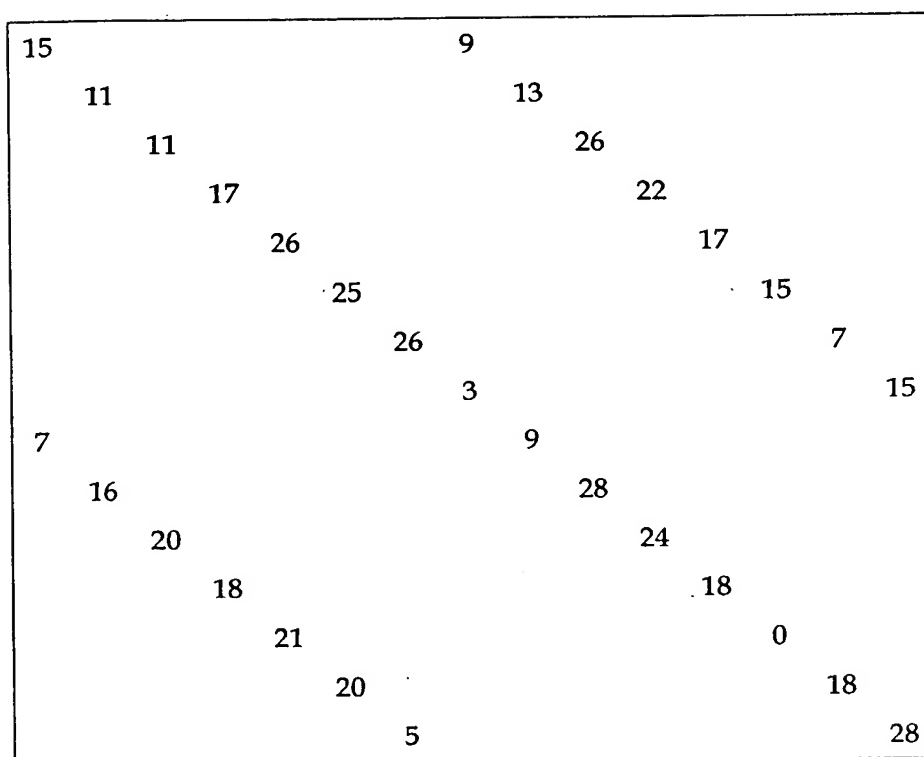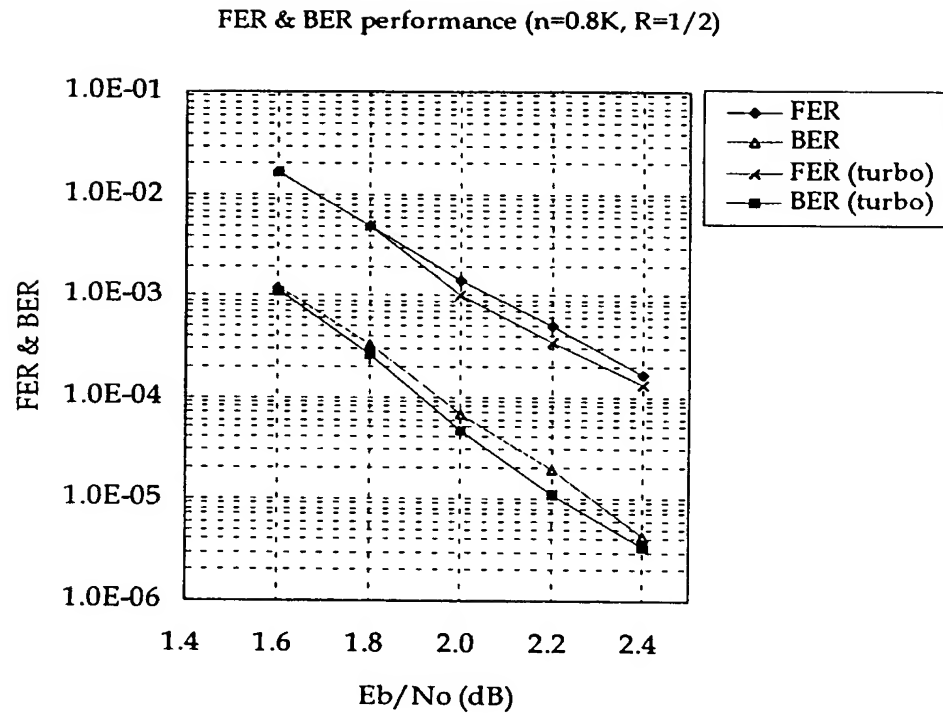
FIG.13C

- H_d

0	0	0	0			0				0				
0	1	2	3			6					11			
0	2	4	6			12					22			
0	3	6	9			18					33			
0	4	8	12				28					48		
0	5	10		20			35					7		
0	6	12		24			42					19		
0	7	14		28				3					38	
0	8	16		32				11					51	
0	9	18		36				19					11	
0	10	20			50				37					34
0	11	22			2				46					48
0	12	24			7				2					9
0	13	26			12					24				
0	14	28			17					34				

FIG.14A

- **H**$_p$

```
15                              37
    1                              46
       28                            33
          47                      13
             49                14
                37          25
                   28        9
                      14        31
12                      44
   28                35
      8                28
         28              4
            41            40
               20          35
                  38          13
```

FIG.14B

FER & BER performance (n=1.6K, R=1/2)
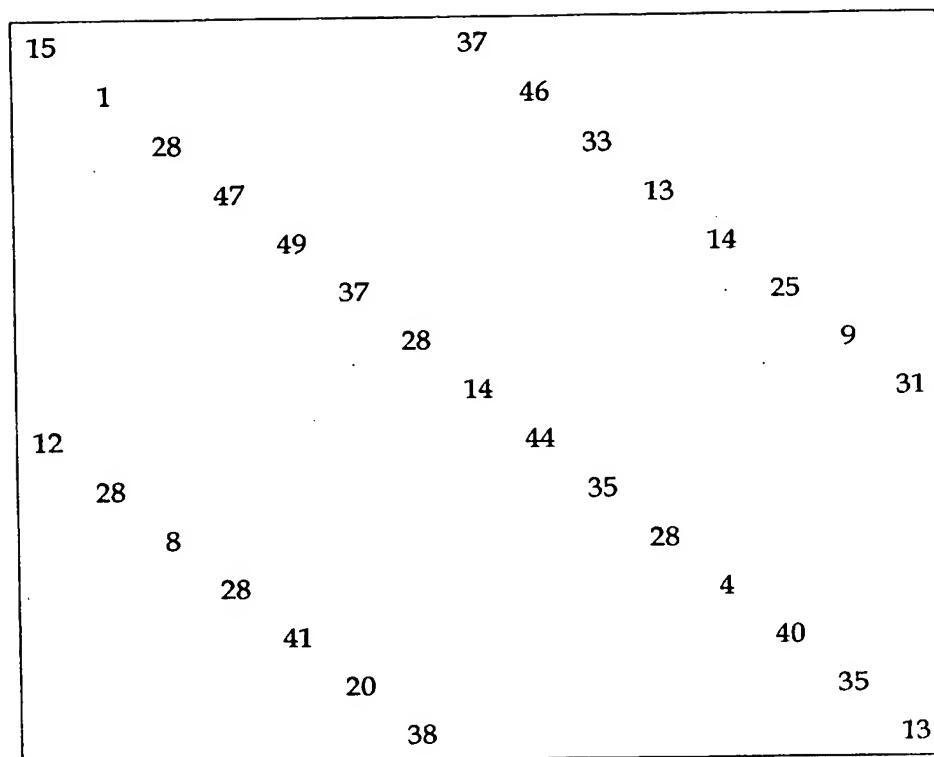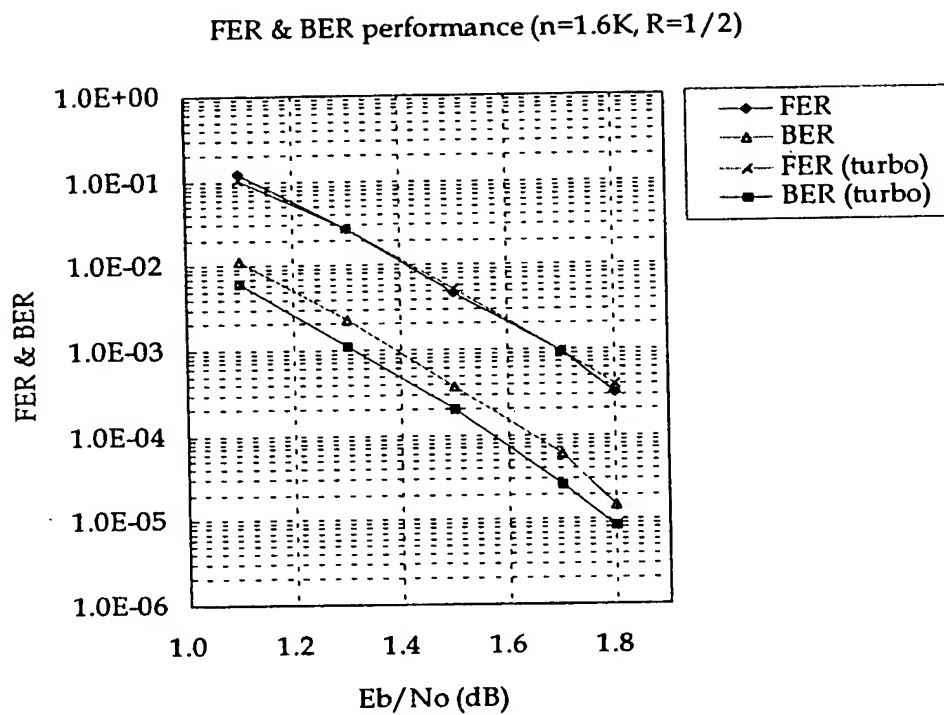
FIG.14C

$- H_d$

0	0	0	0		0		0		
0	1	2	3		6			11	
0	2	4	6		12			22	
0	3	6	9		18			33	
0	4	8	12			28		48	
0	5	10		20		35		60	
0	6	12		24		42		72	
0	7	14		28			56		91
0	8	16		32			64		1
0	9	18		36			72		14
0	10	20			50		90		37
0	11	22			55		99		51
0	12	24			60		5		65
0	13	26			65		27		
0	14	28			70		37		

FIG.15A

- H_p
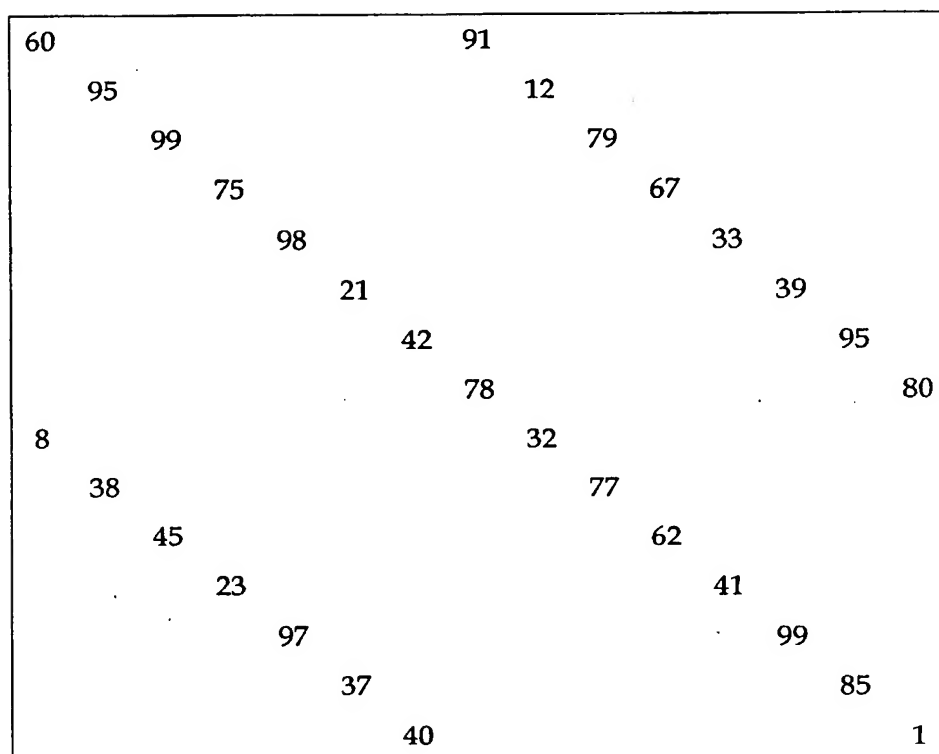
FIG.15B

FER & BER performance (n=3.1K, R=1/2)
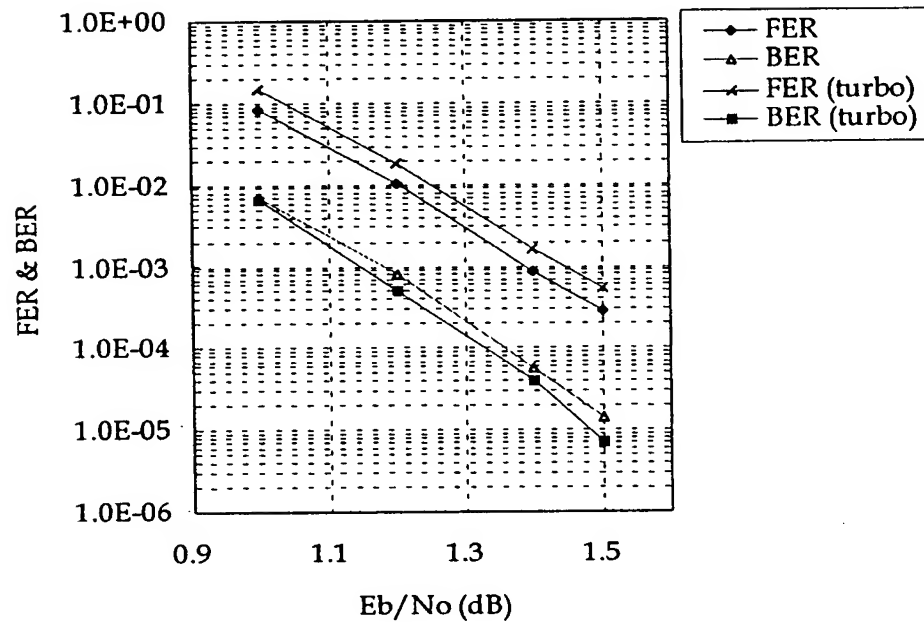
FIG.15C

- **H**$_d$

0	0	0	0		0		0		
0	1	2	3		6			11	
0	2	4	6		12			22	
0	3	6	9		18			33	
0	4	8	12			28			48
0	5	10		20		35			60
0	6	12		24		42			72
0	7	14		28			56		91
0	8	16		32			64		104
0	9	18		36			72		117
0	10	20			50		90		140
0	11	22			55		99		154
0	12	24			60		108		168
0	13	26			65			130	
0	14	28			70			140	

FIG.16A

0	0	0	0			0				0				
0	1	2	3			6					11			
0	2	4	6			12					22			
0	3	6	9			18					33			
0	4	8	12				28					48		
0	5	10		20			35					60		
0	6	12		24			42					72		
0	7	14		28				56					2	
0	8	16		32				64					15	
0	9	18		36				72					28	
0	10	20			50				1					51
0	11	22			55				10					65
0	12	24			60				19					79
0	13	26			65					41				
0	14	28			70					51				

Example of H_d with irregular distribution of $d_v = 15$ (p=89)
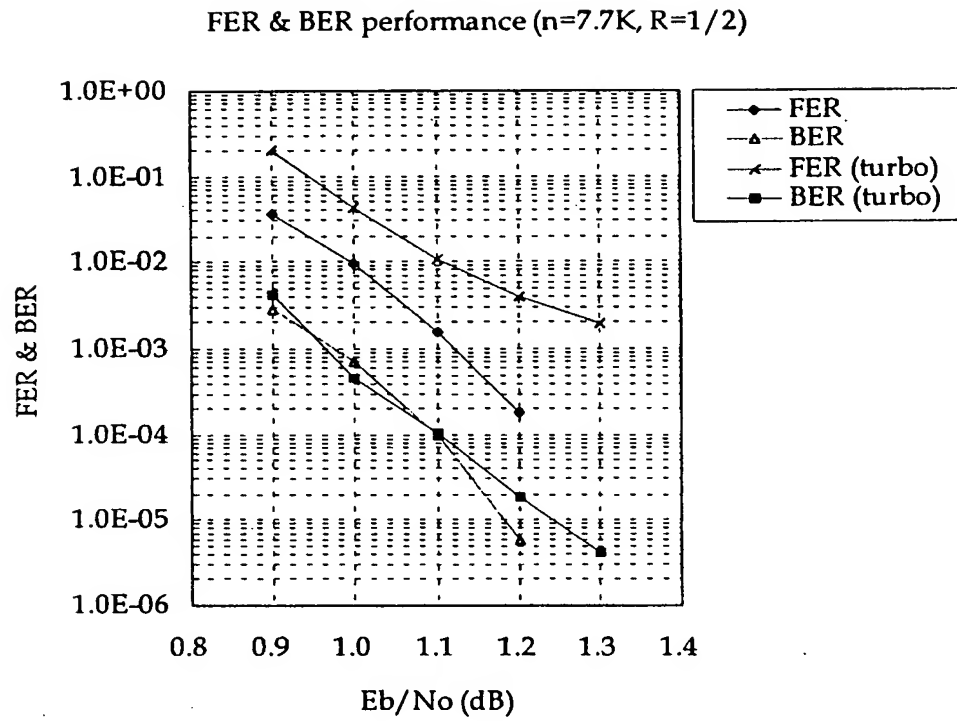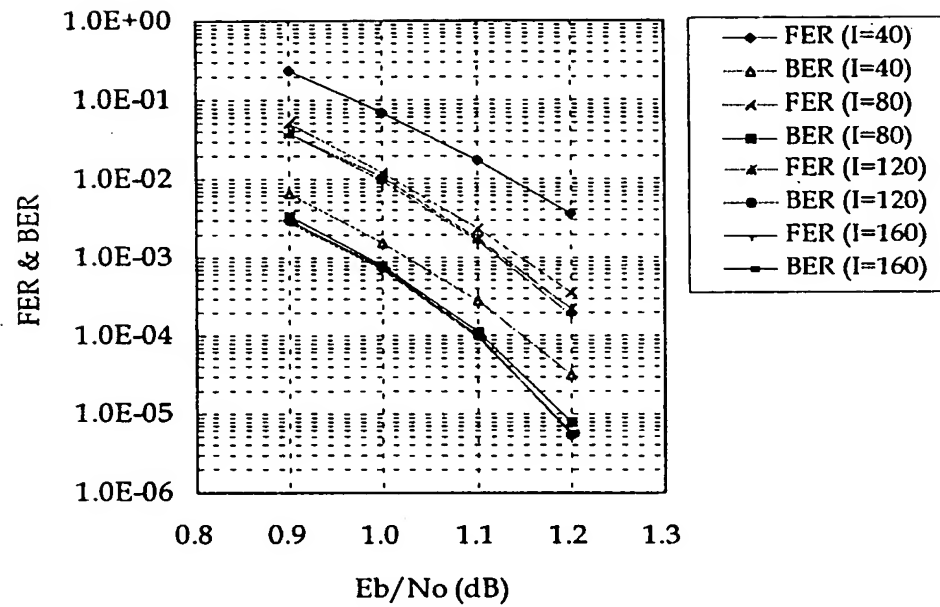
FIG.16B

FER & BER performance (n=7.7K, R=1/2)

FIG.16C

FER & BER performance (n=7.7K, R=1/2)

FIG.16D